UNCERTAINTIES IN THE DYNAMIC BEHAVIOUR OF TIMBER FOOTBRIDGES CONSIDERING HUMAN STRUCTURE INTERACTION

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Keywords: Uncertainty, Timber footbridge, Dynamic behaviour, Human Structure Interaction.

Abstract. A dynamic study of timber footbridges with uncertain mechanical properties considering the Human Structure Interaction (HSI) induced by the pedestrians is presented in this paper. These structural systems made of timber are increasingly employed due to the high stiffness/weight ratio that wood exhibits in relation to others structural materials. More, the development and implementation of laminated beams permits larger spans. These features can lead to lightweight structural systems in which the acceleration levels can exceed the human comfort limits. The sources of uncertainty of this structural model are the timber mechanical and physical properties, Modulus of Elasticity (MOE) and mass density. Also, the geometrical design of the boards that compose the laminated timber beams supporting the floor involves variability in the distances between finger joints. Probability Density Functions (PDFs) of the timber properties are formulated from the Principle of Maximum Entropy (PME). The finger joints distance generates the lengthwise variability of the MOE and mass density functions in each board of the laminated beams. The influence of these stochastic variables in the structural response on a free vibration problem that includes the HSI induced by the human walking is assessed. Pedestrians arrive to the footbridge under a Poisson distribution and the arrival velocity is such that a medium/low transit density is achieved in accordance with the footbridge dimension. Stochastic properties of the HSI model are introduced through their PDFs. Changes in the natural frequencies and damping of the structure induced by the HSI are numerically obtained through the Finite Element Method (FEM) and Monte Carlo Simulations (MCS). These modifications are evaluated in relation to the footbridge occupancy at each instant. The present stochastic model contributes to obtain a more realistic description of the response of this type of structures.

1 INTRODUCTION

Footbridges are one of the most common timber structures. The development and implementation of laminated beams and the high stiffness/weight ratio of this material in comparison to other construction materials allow the covering of rather long spans. In this work, the complete structure is made of Argentinian *Eucalyptus grandis*, one of the most important renewable species cultivated in Argentina. This specie is one of those accepted for structural use by the Argentinean Standard of Timber Structures CIRSOC 601. A method of visual strength grading of sawn pieces of these species has been developed by Piter (2003).

Structural timber is characterized by a considerable variability of its mechanical properties within and among pieces. Then, a stochastic approach becomes necessary in order to attain a more realistic structural model. The stochastic approaches are derived from the probabilistic theories of random variables and processes. A probabilistic model of timber structures where the Modulus of Elasticity (MOE) is represented as a random variable with a lognormal PDF and the mass density through a random variable with normal distribution, both assuming a homogeneous value within a structural element is presented in Köhler et al. (2007). Also, in Kandler and Füssl (2017) a probabilistic model for Glued Laminated Timber (GLT) is proposed using a random process model for the stiffness distribution within each lamination. Brandner and Schickhofer (2015) use probabilistic models for the MOE and the shear modulus considering serial and parallel systems to represent timber elements. The stochastic modeling of Argentinean *Eucalyptus grandis* timber beams is reported by García et al. (2016) and García and Rosales (2017). In these works, the MOE and the mass density are modeled through a gamma random variable.

Timber footbridges must satisfy strength and serviceability requirements. Generally, due to their low weight, the serviceability requirements in terms of peak accelerations constitute the most restrictive condition in their design. An extensive literature review and state of the art report of the dynamic behavior of footbridges was presented in Živanović et al. (2005). Among other topics, the loads models, standards requirements and studies of the walk of people and crowds are reported. The analysis of the unrestricted pedestrian traffic over footbridges with the consideration of Poisson arrival process (Piccardo and Tubino, 2009) and the randomness of the human walking through stochastic load models can be found in several works (Živanović et al., 2010; Živanović, 2012). There exists a great number of uncertain parameters involved in the human walking, which are best represented in a probabilistic framework. A recent approach include the effect of the Human Structure Interaction (HSI) (Venuti et al., 2016; Caprani and Ahmadi, 2016). The HSI model introduce the pedestrian stiffness, mass and damping to the structural model. Then, the coupled system change its dynamical properties.

In this work a dynamic study of a timber footbridge (García et al., 2017) with uncertain mechanical properties considering the HSI induced by the pedestrians is presented. The sources of uncertainty of this structural model are the timber mechanical and physical properties, MOE and mass density. Also, the geometrical design of the boards that compose the laminated timber beams supporting the floor involves variability in the distances between finger joints. Probability Density Functions (PDFs) of the timber properties are formulated from the PME. The finger joints distance generates the lengthwise variability of the MOE and mass density functions in each board of the laminated beams. The influence of these stochastic variables in the structural response on a free vibration problem that includes the HSI induced by the human walking is assessed. Pedestrians arrive to the footbridge under a Poisson distribution and the arrival velocity is such that a medium/low transit density is achieved in accordance with the footbridge dimension. Stochastic properties of the HSI model are introduced through their PDFs. Changes in the natural frequencies and damping of the structure induced by the HSI are numerically obtained through the Finite Element Method (FEM) and Monte Carlo Simulations (MCS). These modifications are evaluated in relation to the footbridge occupancy at each instant. The present stochastic model contributes to obtain a more realistic description of the response of this type of structures.

2 FOOTBRIDGE DESCRIPTION

The footbridge (García et al., 2017) is composed of three longitudinal, simply supported, laminated beams with a length of 13.2 m and a separation of 0.6 m in the transversal direction of the structure, five transversal laminated beams with a separation of 3.3 m in the longitudinal direction of the structure and a deck of timber boards. The width of the laminated beams and the timber boards of the deck is fixed in 0.15 m, and the height of each lamina of the beams and of the timber boards, in 0.0375 m. The number of layers of the beams is considered between 8 and 16; laminated beams with 0.3 m and 0.6 m of height, respectively.

3 STRUCTURAL TIMBER

3.1 Elastic model

The material model is derived from the transversal isotropic model with two main directions the longitudinal also named parallel to the main fibres direction $E_L = E_x$, and the perpendicular direction that includes the radial and tangential material direction $E_R = E_T = E_{yz}$. The basic stochastic properties proposed in this work are the longitudinal MOE (E_x) and the mass density (ρ). In what follows, these random variables will be represented by $\mathbb{E}_x$ and $\mathbb{P}$, respectively. For a transversally isotropic material, the elastic and shear modulus are defined as $\mathbb{E}_{zy} = \mathbb{E}_x/15$, $\mathbb{G}_{xy} = \mathbb{G}_{xz} = \mathbb{E}_x/16$. Meanwhile, in a general form, the poison coefficients for hardwood are $\nu_{RT} = 0.67, \nu_{LT} = 0.46$ and $\nu_{LR} = 0.39$ (Argüelles Álvarez R, 2013). For a transversally isotropic formulation, $\nu_{zy} = \nu_{RT} = 0.67$, $\nu_{xzy} = (\nu_{LT} + \nu_{LR})/2$ and $\mathbb{G}_{zy} = \mathbb{E}_{zy}/2(1 + \nu_{zy})$.

3.2 MOE and mass density stochastic representation

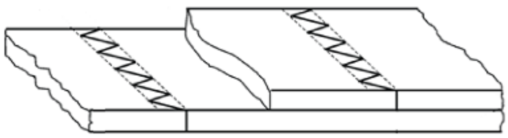
If a stochastic approach is applied to this problem, first a Probability Density Function (PDF) should be chosen for the random variable. A statistical concept of entropy was introduced by Shannon (1948) and its maximization, by Jaynes (1957) . The Principle of Maximum Entropy (PME) states that, subjected to known constrains, the PDF which best represents the current state of knowledge is the one with largest entropy. It is possible to demonstrate that the application of the principle under the constraints of positiveness and bounded second moment, leads to a gamma PDF. The PME conduces to this PDF due to the fact that the domain of both the MOE and the mass density is real and positive. To find the parameters of the marginal PDF of the MOE and mass density, experimental data presented by Piter (2003) were employed. The parameters of the gamma marginal PDFs of the MOE and density are estimated with the help of the Maximum Likelihood Method (MLM). Then, the Kolmogorov-Smirnov (K-S) and the Andersson-Darling (A-D) test of fit are used, (Benjamin and Cornell, 1970). The PDF that best fits with the experimental values of the MOE is the gamma, in coincidence with the PME result. The test of fit was also carried out with the lognormal, normal and truncated normal PDFs. On the other hand, for the density, the four PDFs fulfill the critical value, but the lognormal and gamma fit best with respect to the experimental values. Here, following the PME and due to the small difference found among the lognormal and gamma, the last PDF is adopted to represent the mass density uncertainty. The gamma marginal CDF of the MOE and mass density writes:

$$F(x \mid a, b) = \frac{1}{b^a \Gamma(a)} \int_0^x t^{a-1} e^{-\frac{t}{b}} dt \tag{1}$$

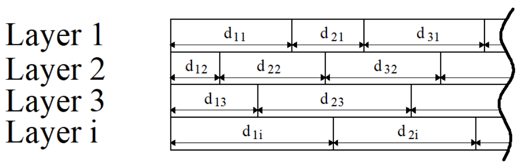
where a and b denote the shape and scale parameters, respectively. For the MOE, the parameters are a = 34.582 and b= 0.402 with a mean value of the MOE equal to 13.902 GPa and a standard deviation of 1.498 GPa. In the case of the mass density, a = 72.179 and b = 7.659, the mean value of the mass density equal to 552.819 kg/m^3 and a standard deviation of 65.069 kg/m^3.

3.3 Laminated beams

Laminated beams are composed of several layers each formed by the union of boards with different mechanical properties. The upper and lower faces of the boards are glued to the superior and inferior continuous board. Previously, the boards of each lamina are assembled together by finger joints unions. Distances between two finger joints obtained from a visual survey of laminated beams were employed in order to simulate the different boards that conform a laminated beam. With the results of the survey, the Probability Mass Function (PMF) of the distance between fingers joints was constructed. The mean value and standard deviation of the distance between joints are 0.865 m and 0.247 m, respectively. According to the Argentinean standard IRAM:9662-2 (2006) , each board of the laminated beams comes from a specific strength class. Within this quality class, the properties vary stochastically. The finger joint union is shown in Fig. 1(a) and an illustration of the distances between consecutive unions at each layer obtained from the PMF, in Fig. 1(b).



(a) Finger joints, IRAM:9660-2.



(b) Finger joint unions distribution according to the PMF.

Figure 1: Laminated beams design.

4 FINITE ELEMENT DISCRETIZATION

The natural frequencies and modes of the footbridge are obtained solving the Eq. (2) below:

$$[\mathbf{K} - V_n^2 \mathbf{M}]\Phi_n = 0 \tag{2}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the n x n positive-definite global stiffness and mass matrices, respectively. The equation of motion was discretized for laminated beams using Timoshenko beam elements with two nodes and three degrees of freedoms per node. Translational, rotational and torsional degrees of freedom, were employed (Reddy, 1993). The torsional stiffness was obtained from Swanson (1998). For the deck of the footbridge, rectangular bilinear plate elements with four nodes and three degrees of freedom per node (Reddy, 1993) were employed.

5 COUPLED SYSTEM HUMAN STRUCTURE

5.1 Beam model of the footbridge

After the modal study of the whole structure a reduction of the same was carried out in order to represent the footbridge as an equivalent simply supported beam for the application of the Human Structure Interaction (HSI) model. The properties of the equivalent beam model of the footbridge are depicted in Table 1.

	8 layers		12 layers		16 layers	
	Mean	SD	Mean	SD	Mean	SD
F_1 Hz	3.22	0.02	5.00	0.03	6.77	0.03
EI_b Nm^2	14.17 x10^6	0.22 x10^6	47.45 x10^6	0.58 x10^6	111 x10^6	1.19 x10^6
m kg/m	110.82	0.58	153.80	0.70	196.78	0.78

Table 1: Properties of the equivalent beam model of the footbridge.

The Damping of the footbridge C_b was modelled through a truncated exponential PDF with mean value equal to 3% and a range between 1% and 7% proportional to the stiffness and mass.

5.2 Spring Mass Damper (SMD) model for the Human Structure Interaction (HSI)

In this model, the pedestrian is modelled as a single degree of freedom system with damping c_p, mass m_p and stiffness k_p (Caprani and Ahmadi, 2016). Then, the equation of motion for the pedestrian is given by:

$$m_p\ddot{y} + c_p\left(\dot{y} - \dot{w}\right) + k_p\left(y - w\right) = 0 \tag{3}$$

where y is the displacement of the human mass from the equilibrium position an w is the beam deflection. The equation of motion of the beam is the following:

$$m_b\ddot{w} + c_b\dot{w} + k_b w = G(t) \tag{4}$$

where m_b, c_b and k_b are the mass, damping and stiffness of the beam. For the Finite Element discretization the pedestrian travels with constant velocity v. The interaction force between the SMD system and the beam is:

$$f\left(x,t\right) = G\left(t\right) + c_p\left[\dot{y} - \dot{w}\right] + k_p\left[y - w\right] \tag{5}$$

Using the shape function for the displacement of the beam:

$$\dot{w}\left(x,t\right) = v\mathbf{N_x d} + \mathbf{N\dot{d}} \tag{6}$$

we obtain:

$$f\left(x,t\right) = G\left(t\right) + c_p\dot{y} + k_p y - c_p v\mathbf{N_x d} - c_p\mathbf{N\dot{d}} - k_p\mathbf{Nd} \tag{7}$$

The equation of motion of the pedestrian becomes:

$$m_p\ddot{y} + c_p\dot{y} + k_p y - c_p\mathbf{N\dot{d}} - \left(c_p v\mathbf{N_x} + k_p\mathbf{N}\right)\mathbf{d} = 0 \tag{8}$$

and similarly, the equation of motion for the beam:

$$\mathbf{M_b}\ddot{\mathbf{d}} + \left(\mathbf{C_b} + c_p\mathbf{N^T N}\right)\dot{\mathbf{d}} + \left(\mathbf{K_b} + c_p v\mathbf{N^T N_x} + k_p\mathbf{N^T N}\right)\mathbf{d} - c_p\mathbf{N^T}\dot{y} - k_p\mathbf{N^T}y = \mathbf{N^T}G(t) \tag{9}$$

These two coupled equations can be expressed as:

$$\begin{bmatrix} \mathbf{M_b} & 0_{Nx1} \\ 0_{1xN} & m_p \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{d}} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} \mathbf{C_b} + c_p\mathbf{N^T N} & -c_p\mathbf{N^T} \\ c_p\mathbf{N} & c_p \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{d}} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_b} + c_p v\mathbf{N^T N_x} + k_p\mathbf{N^T N} & -k_p\mathbf{N^T} \\ -c_p v\mathbf{N_x} - k_p\mathbf{N} & k_p \end{bmatrix} \begin{Bmatrix} \mathbf{d} \\ y \end{Bmatrix} = \begin{Bmatrix} \mathbf{N^T}G(t) \\ 0 \end{Bmatrix} \tag{10}$$

and then:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{P} \tag{11}$$

Whit the application of the principle of effect superposition, the formulation for a simple pedestrian can be extended to multiple pedestrian transit (Caprani and Ahmadi, 2016). When the pedestrian crosses the footbridge, the coupled system (human-structure) changes its dynamic properties. For the modal analysis of the system the state space method is employed in order to obtain the modal properties. The system equation in the state space can be written as:

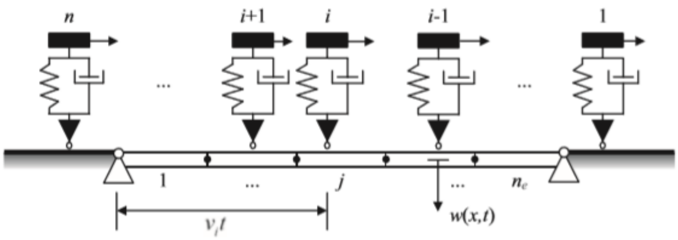


Figure 2: Spring, Mass, Damper (SMD) for the Human Structure Interaction (HSI) (Caprani and Ahmadi, 2016).

$$\dot{\mathbf{v}} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M^{-1}K} & -\mathbf{M^{-1}C} \end{bmatrix} \begin{Bmatrix} \mathbf{d} \\ \dot{\mathbf{d}} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\mathbf{M^{-1}P}(t) \end{Bmatrix} \tag{12}$$

in which $\mathbf{I}$ is the identity matrix and $\mathbf{P}(t)$ is the force vector. Solving the eigenvalue problem the modal properties are obtained:

$$\mathbf{A}\phi = \lambda\phi \tag{13}$$

where λ is the complex eigenvalue and ϕ is the corresponding eigenvector. The natural frequency f_i and the damping ratio ξ_i are obtained from:

$$f_i = \frac{|\lambda_i|}{2\pi}; \quad \xi_i = \frac{|Re\,(\lambda_i)|}{|\lambda_i|} \tag{14}$$

5.3 Stochastic properties of the SMD model

The stochastic variability of the pedestrian properties in the HSI model (m_p, c_p, and k_p) have been considered following Venuti et al. (2016), (Table 2):

	Pedestrian mass (m_p) kg	Pedestrian damping (c_p) Ns/m	Pedestrian stiffness (k_p) N/m
PDF	Normal, Mean=75kg, SD=15 kg	Uniform [0, 400]	Uniform [2000, 13000]

Table 2: Probability Density Functions (PDF) of the pedestrian properties in the HSI model.

5.4 Transit of multiple pedestrians

In order to simulate the transit of multiple pedestrians, the arrival time is simulated through a Poisson process. This distribution was experimentally confirmed in real footbridges in Matsumoto (1978).The times between arrivals (A_i) have an exponential distribution of parameter λ_0, simulated as $A_i = -(1/\lambda_0)\ln U_i$ in which U_i are random values from an uniform distribution between $[0, 1]$ (Rubinstein and Kroese, 1981). Then, the arrival times $T_i = A_1 + \cdots + A_i$ are simulated within the analysis time $[0, T]$ in which T is equal to 30 s and the mean arrival velocity λ_0 is equal to 0.2 pedestrians/s. The mean arrival velocity λ_0 was adopted in order to ensure a low pedestrians density according to the dimensions and the use of the footbridge analyzed in this study. A limit of low occupancy density is considered equal to 0.6 pedestrians /m^2 FIB (2005). For this limit or lower values, the individuals are capable to walk freely keeping their transit characteristics (step frequency and length).

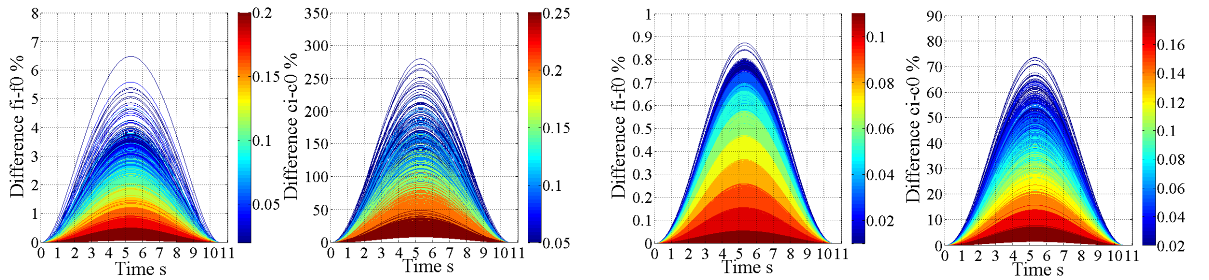
6 NUMERICAL RESULTS

6.1 Single pedestrian

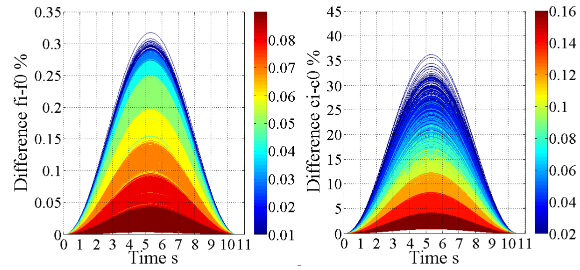
In Fig. 3, modifications in the first natural frequency and modal damping of the coupled system Human-Structure regarding the initial footbridge modal properties when a single pedestrian cross the structure are shown. After a converge study a number of 5000 independent Monte Carlo simulations was carried out. As can be viewed, the modification in the first modal damping is higher than in the first frequency of the coupled system. When the stiffness of the laminated beams increase, the change in the first natural frequency and in the damping decrease. In the figures, the color pattern indicates the relative frequency of each percentage change. An important modification in the modal damping is observed when the laminated beams of the footbridge are composed of 8 layers. In Fig. 4 an approximation of the relation between the initial footbridge modal damping C_b, the pedestrian damping C_p and the maximum value of the modal damping of the coupled system registered for each one of the simulations is shown. In the figures the color pattern indicates the maximum value of the coupled system damping when the pedestrian walk along the footbridge. As can be observed in Fig. 4(a), for laminated beams with 16 layers, the change in the damping is lower and can be approximated with a linear relation. Additionally, when the laminated beams are composed for 8 layers (Fig.4(b)), the change in the damping is more important and also can be approximated with a linear relation. For the figure, one can infer that the maximum differences between the initial footbridge modal damping C_b and the maximum value of the modal damping of the coupled system in Fig. 3, correspond to footbridges with low initial modal damping C_b.

6.2 Multiple pedestrian

In Fig. 5 the percentage differences between the initial modal damping of the footbridge and the maximum modal damping of the coupled system in relation to the footbridge occupancy are registered for the case of multiple pedestrian transit in 5000 simulations. In Fig. 5(a) for an arrival velocity of 0.1 pedestrians/s, in Fig. 5(b) for an arrival velocity of 0.2 pedestrians/s and

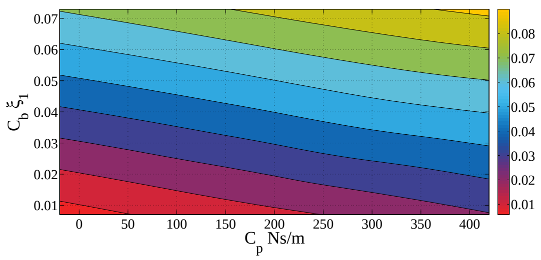


(a) 8 layers. (b) 12 layers.

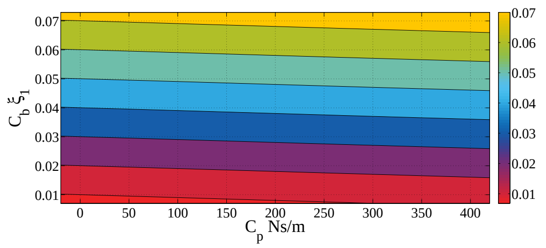


(c) 16 layers.

Figure 3: Variation in the first natural frequency and modal damping of the footbridge for different number of layers in the laminated beams. Left plots: variation of the natural frequency. Right plots: variation of the modal damping. Color pattern: results frequency.



(a) 8 layers.



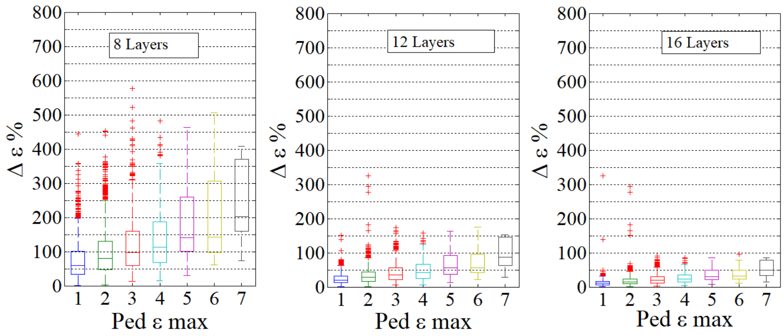
(b) 16 layers.

Figure 4: Approximation of the relation between the initial footbridge modal damping C_b, the pedestrian damping C_p and the maximum value of the modal damping registered for each one of the simulations.

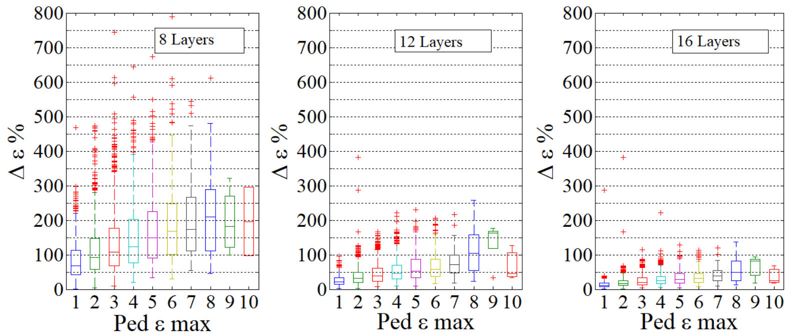
in Fig. 5(c) for an arrival velocity of 0.3 pedestrians/s. As can be seen the arrival velocity do not modify the damping differences in a considerable way. Only a modification in the conditions in which the percentage differences occurs related with the footbridge occupancy can be appreciated. For each arrival velocity, the variation in the beam stiffness constitutes the principal source of damping variation in the coupled system Human-Structure.

7 CONCLUSIONS

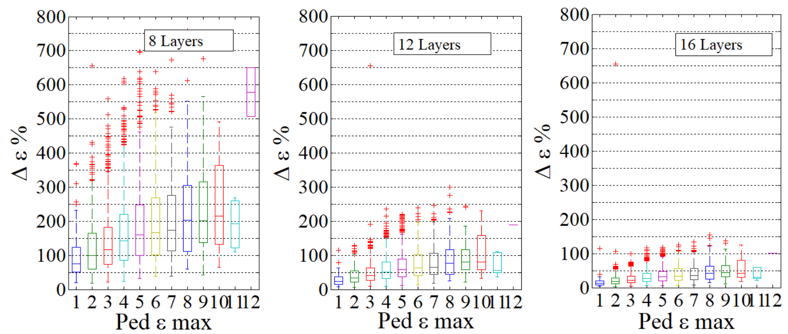
A stochastic study of the modal dynamic behaviour of a short span timber footbridge made of Argentinean *Eucalyptus grandis* considering the Human Structure Interaction (HSI) was presented. To the best knowledge of the authors', this structural configuration is extensively employed but not adequately studied in the literature. The stochastic analysis allows to extend the range of the response starting from an equivalent beam model of the footbridge. When the stiffness of the laminated beams increase the change in the first natural frequency and in



(a) λ=0.1 pedestrians/s.



(b) λ=0.2 pedestrians/s.



(c) λ=0.3 pedestrians/s.

Figure 5: Percentage differences in the modal damping. Multiple pedestrian transit.

the damping decrease. An important modification in the modal damping is observed when the laminated beams of the footbridge are composed of 8 layers. The change in the damping is more important and also can be approximated with a linear relation. As can be seen the arrival velocity do not modify the damping differences in a considerable way. Only a modification in the conditions in which the percentage differences occurs related with the footbridge occupancy can be appreciated. For each arrival velocity, the variation in the beam stiffness constitutes the principal source of damping variation in the coupled system Human-Structure. The present stochastic model contributes to obtain a more realistic description of the response of this type of structures.

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